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Classical, fuzzy, hesitant fuzzy and intuitionistic fuzzy analytic hierarchy processes applied to industrial maintenance management

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Abstract. A multi-criteria problem involves the consideration of two or more criteria in the prioritization of alternative solutions. The Analytic Hierarchy Process (AHP) is a leading multi-criteria method. Consistency checking is a great advantage of AHP. Since in AHP priorities come from pairwise comparisons, it is possible to check the consistency of these comparisons. However, a problem occurs when comparisons fail the consistency check. Then, the excluding options are to review some comparisons (Option 1) or to keep the comparisons (Option 2). This paper presents an AHP application in the maintenance management of an industrial plant. Industrial maintenance is not in the core business of an organization. However, maintenance costs can account over 50% of production costs. One of the first maintenance management decisions is on the maintenance strategy. Shall maintenance anticipate the occurrence of failure? Or shall maintenance be performed after an equipment breakdown? Answering those questions with classical AHP resulted in inconsistent comparison matrices. In that case, Fuzzy AHP (FAHP) were applied, avoiding this situation. Therefore, the purpose of this paper is to present the applications of four AHP models: Classical AHP and three models of FAHP, including hesitant fuzzy sets and intuitionistic fuzzy sets. The application of Hesitant FAHP (HFAHP) and Intuitionistic FAHP (IFAHP) are the novelty of this paper. The four AHP models were also applied in the same case of maintenance management of an industrial plant. Results were very similar, but experts could express their preferred model.

Keywords: Analytic hierarchy process, hesitant fuzzy sets, intuitionistic fuzzy sets, maintenance management

1. Introduction

A decision problem is a problem with more than one alternative solution. With multi-criteria decision making (MCDM), alternatives are identified or prioritized considering two or more criteria. The Analytic Hierarchy Process (AHP) is a leading MCDM method [1].

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AHP was named due the use of hierarchies to represent an MCDM problem [2, 3]. A three-level hierarchy has the decision objective in the top level, criteria in the middle and alternatives in the bottom level. AHP was originally proposed as a method to measuring the fuzziness of sets [4]. In AHP, experts provide comparisons between alternatives and between criteria. Priorities are obtained with the right eigenvector of a pairwise comparisons matrix. Quality of comparisons is checked with matrix consistency. The Consistency Index (CI) and the Consistency Ratio (*CR*) are indicators based on the eigenvalue of the matrix. Generally, comparisons may be accepted if $CR \leq 0.1$ [5].

What if a comparisons matrix has $CR > 0.1$? There are two excluding options. Option 1 is to accept priorities, even though generated by inconsistent pairwise comparisons. Option 2 is to review some or all comparisons in inconsistent matrices. At first, Option 1 seems illogical, because it implies disregarding the consistency check. Therefore, some intelligent techniques, such as genetic algorithms, have been applied to Option 2 [6].

Sometimes review comparisons seem not to be the best way. This happens mainly when some effort was made to collect comparisons from experts. Asking them to review their comparisons may be embarrassing, at first. Nevertheless, $CR > 0.1$ means that comparisons are not logically connected. Fuzzy sets are indicated in situations where experts are not confident about their comparisons. Some uncertainty may be involved. Previously, a simple Fuzzy AHP approach was applied in an ex-post facto case study [7]. In comparison with the original AHP application, the results were much the same. However, experts were more confident with the process of making comparisons with triangular fuzzy sets (TFS). Now, this paper advances this study with more sophisticated fuzzy sets. Therefore, Hesitant Fuzzy Analytic Hierarchy Process (HFAHP) and Intuitionistic Fuzzy Analytic Hierarchy Process (IFAHP) were applied in the same case: maintenance management of an industrial plant. The application of HFAHP and IFAHP is the main novelty of this paper, since they were recently introduced in the international literature [8, 9].

Section 2 presents a literature review, justifying the originality and novelty of this paper. Section 3 presents the AHP, FAHP, HFAHP, and IFAHP methods. Section 4 presents the applications of those methods in the case of industrial maintenance management. Section 5 brings conclusions and proposals for future research, followed by acknowledgements and references.

2. Literature review

Besides strategic implications, literature on AHP, fuzzy sets and maintenance management is scarce. Searching the Scopus database with keywords "AHP", "fuzzy" and "maintenance" resulted in only 72 documents, including conference papers and journal articles.

Table 1
Publication on AHP, fuzzy sets and maintenance management. Source: Scopus

Keywords	Documents
"AHP" AND "fuzzy" AND "maintenance"	72
"AHP" AND "fuzzy"	3,664
"AHP" AND "maintenance"	272
"fuzzy" AND "maintenance"	1,058
"AHP"	83,068
"fuzzy"	250,063
"maintenance"	1,657,446

The Web of Science database was not searched, first because it does not provide open access. According to the literature on Bibliometrics, contents in both database are quite similar [10]. In May 2019, date of the search, each database had over 60 million documents.

Table 1 presents the impact of removing one or two keywords. Removing "maintenance", there are over 3,500 documents on AHP and fuzzy sets. Re-adding "maintenance" and removing "AHP", over a thousand documents on fuzzy sets and maintenance management. Re-adding "AHP" and removing "fuzzy", only 272 documents on AHP and maintenance management.

The relative scarcity of works involving the three keywords, simultaneously, is evidence of the originality of this paper. In other words, documents on AHP, fuzzy sets and maintenance management represent under 0.03% of the total documents on fuzzy sets. Therefore, this paper makes a contribution to the AHP, fuzzy sets and maintenance management literature.

Fuzzy sets were introduced to deal with the uncertainty due to imprecision and vagueness [11]. Figure 1 presents the evolution of fuzzy sets theory [12]: from original type-1 fuzzy sets [13], through type-n fuzzy sets [14] and interval-valued fuzzy sets [15], followed by intuitionistic fuzzy sets [16], fuzzy multisets [17], non-stationary fuzzy sets [18], and finally, hesitant fuzzy sets [19]. The novelty of this paper comes with the application of HFAHP and IFAHP. These models of AHP are based on the latest developments of fuzzy sets theory.

3. Methodology

This section presents concepts and equations for AHP (Subsection 3.1), FAHP (Subsection 3.2), IFAHP (Subsection 3.3), and HFAHP (Subsection 3.4).

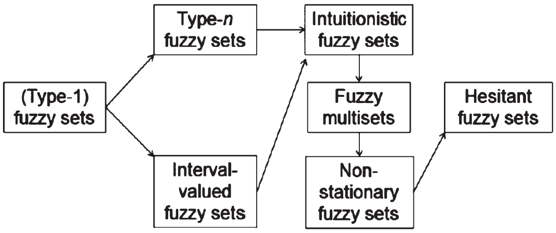

Fig. 1. Setting for document template.

3.1. Analytic hierarchy process

In AHP, priorities for alternatives and criteria are obtained with the right eigenvector w of a comparisons matrix $\boldsymbol{A}$, as in Equation 1, where $\lambda_{\max}$ is the maximum eigenvalue.

$$\boldsymbol{A}w = \lambda_{\max}\boldsymbol{w} \tag{1}$$

For fully consistent matrices, $a_{ik} = a_{ij}a_{jk}$ and $\lambda_{\max} = n$ [20]. Otherwise, $\lambda_{\max} > n$. The closer $\lambda_{\max}$ is to n, the more $\boldsymbol{A}$ can be considered consistent [21]. The Consistency Index *CI* (Equation 2) is a better measure for the consistency, since *CI* compares the deviation between $\lambda_{\max}$ and n with the degree of freedom $n - 1$.

$$CI = \frac{\lambda_{\max} - n}{n - 1} \tag{2}$$

The Consistency Ratio $RC = CI/RI$, where *RI* is the Random Index [3], is a more complete parameter for the consistency of a comparison matrix. *CR*'s upper limit is 0.1 [10].

When more than one expert is available, they can provide individual comparisons matrices. If they are from the same company or department, that is, if they share common interest, their comparisons must be aggregated before w is computed. Saaty and Peniwati [22] suggested the aggregation of individual judgments by their geometrical mean.

3.2. Fuzzy analytic hierarchy process

A fuzzy set $\hat{X}$ is characterized by a membership function $\mu(x)$ which associates each element x to a real number in the interval [0, 1] [13]. Triangular fuzzy sets (TFS) are common membership functions used in engineering, because they are easy to manage [23, 24]. A TFS is often represented by a vector $[l, m, u]$, being m the modal value, that is $\mu(m) = 1$, and l and u being the lower and upper limits, or $\mu(l) = \mu(u) = 0$, as in Equation 3.

$$\mu(x) = \begin{cases} 0 & x < l \\ \dfrac{x-l}{m-l} & l \le x < m \\ \dfrac{u-x}{u-m} & m \le x < u \\ 0 & x \ge u \end{cases} \tag{3}$$

In FAHP, all comparisons and all priorities are TFS: $\hat{a}_{ij} = [l_{ij}, m_{ij}, u_{ij}]$ and $\hat{w}_i = [l_i, m_i, u_i]$. Then, for every a_{ij} experts now need to provide three values l_{ij}, m_{ij} and u_{ij}. Priorities can be estimated as in Equation 4, proposed by [25].

$$\hat{w}_i = \left[\frac{\sum_{j=1}^{n} l_{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{n} u_{ij}}, \frac{\sum_{j=1}^{n} m_{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{n} m_{ij}}, \frac{\sum_{j=1}^{n} u_{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{n} l_{ij}}\right] \tag{4}$$

There are several formulas to defuzzify a TFS. Equation 5 is the simplest one adopted in FAHP.

$$w_i = \frac{l_i + m_i + u_i}{3} \tag{5}$$

3.3. Intuitionistic fuzzy analytic hierarchy process

Intuitionistic fuzzy sets incorporate a degree of hesitation which is defined as 1 minus the sum of membership $\mu(x)$ and non-membership $v(x)$ values [8]. An intuitionistic triangular fuzzy set (ITFS) with $\mu(x)$ as in Equation 3 and $v(x)$ as in Equation 6 is denoted as $[l, m, u; l', m, u']$.

$$v(x) = \begin{cases} 1 & x < l' \\ \dfrac{m-x}{m-l'} & l' \le x < m \\ \dfrac{x-m}{u'-m'} & m \le x < u' \\ 1 & x \ge u' \end{cases} \tag{6}$$

A TFS is a particular case of an ITFS, with $l = l'$ and $u = u'$. In IFAHP, all comparisons and all priorities are ITFS: $\hat{a}_{ij} = [l_{ij}, m_{ij}, u_{ij}; l'_{ij}, m_{ij}, u'_{ij}]$ and $\hat{w}_i = [l_i, m_i, u_i; l'_i, m_i, u'_i]$. Priorities can be estimated as in Equation 7:

Table 2
Publication on AHP, fuzzy sets and maintenance management. Source: [26]

Linguistic term	IFS
Absolutely strong (AS)	[7.5, 9, 10.5; 7, 9, 11]
Very strong (VS)	[5.5, 7, 8.5; 5, 7, 9]
Fairly strong (FS)	[3.5, 5, 6.5; 3, 5, 7]
Slightly strong (SS)	[1.5, 3, 4.5; 1, 3, 5]
Exactly equal (EE)	[1, 1, 1; 1, 1, 1]

$$\hat{w}_i = \begin{bmatrix} \prod_{j=1}^{n} l_{ij}, \ \prod_{j=1}^{n} m_{ij}, \ \prod_{j=1}^{n} u; \\ \prod_{j=1}^{n} l'_{ij}, \ \prod_{j=1}^{n} m_{ij}, \ \prod_{j=1}^{n} u'_{ij} \end{bmatrix} \quad (7)$$

Defuzzification of an ITFS is done by applying Equation 8, based on [26], where α is the highest value of $\mu(x)$. Usually, $\alpha = 1$.

$$w_i = \frac{\frac{(u'_i - l_i) + (u'_i - l_i)}{3} + l'_i + \alpha \left[\frac{(u_i - l_i) + (m_i - l_i)}{3} + l_i \right]}{2} \quad (8)$$

Table 2 presents a triangular fuzzy intuitionistic scale. This scale was proposed by [26], to facilitate IFAHP application.

3.4. Hesitant fuzzy analytic hierarchy process

Hesitant fuzzy sets (HFS) are extensions of fuzzy multisets (FMS). FMS are based in the concept of bags, that is, a set with repeated elements [27]. A FMS has more than one membership value for x. For TFS $[l, m, u]$, a triangular FMS could be $[(0, 0.2)/l, (0.75, 0.8, 1)/m, (0, 0.1)/u]$.

HFS are similar to FMS, since a set of values of memberships are possible for a single element x. A representation of an HFS is $\{\langle x, h(x) | x \in X \rangle\}$, where $h(x)$ denotes a set of membership values.

With more than two experts involved, an envelopment approach [8] improves data collection. It is particularly useful when experts hesitate to express their assessments [18]. Another advantage is that experts need to provide only one value for every a_{ij}.

Let a_{ij}^k be the pairwise comparison, between j and j, provided by expert $k = 1, 2, 3 \ldots K$. A hesitant triangular fuzzy set $\hat{a}_{ij}$ is obtained with Equations 9 to 11.

$$l_{ij} = \min_k a_{ij}^k \quad (9)$$

$$u_{ij} = \max_k a_{ij}^k \quad (10)$$

$$m_{ij} = \frac{\sum_{k=1}^{K} a_{ij}^k - l_{ij} - u_{ij}}{K - 2} \quad (11)$$

4. Case study

4.1. Object of study

The typical industrial plant analyzed here is located in the southeastern Brazilian state of Sao Paulo. This plant belongs to a global manufacturer of components, primarily, for the automobile industry, civil construction and railroad equipment. The plant manufactures structural components for excavators, heavy machines, tractors, and trains. Its main customers and suppliers are also industrial plants belonging multinational groups, located in southeastern Brazil.

At the end of 2018, three managers were involved with maintenance management: A 42-year old electrical engineer with five years of experience in the plant, plus 15 years in another company; a 33-year old technologist in industrial maintenance with four years of experience in the plant; and a 30-year old mechanical engineer with three years of experience in the company.

Despite their different ages, experiences and job titles, their opinions were equally considered, since they all were involved with maintenance management.

Among other responsibilities, managers were in charge on the selection of maintenance strategies for plant's equipment. One of the four following alternatives could be adopted, based only on the managers' common sense:

- Corrective Maintenance (A1) is when the intervention is at the moment, or after, a failure occurs. The failure makes the equipment unavailable, or performs worst. Corrective maintenance can be planned, or not. Often, when unplanned, the intervention is immediate and without service preparation. When planned, the intervention team can be prepared for the intervention.
- Preventive Maintenance (A2) is when the intervention is carried out to reduce or avoid the failure of the equipment. The interventions follow a previously prepared plan, and they are periodically carried out.
- Predictive Maintenance (A3) is when the intervention modifies parameters of performance. It

prevents failures by means of various parameters, which aim to assure the continuous operation of the equipment for the maximum possible time.
– Proactive Maintenance (A4) is based on the frequency of occurrence of the failure. A history of these occurrences is recorded for the equipment and the information is used to determine the root cause of failure. It generates actions related to the root cause of failure, seeking to increase the life of the equipment.

Based on the literature on maintenance management and in company's key performance indicators (KPI), five criteria were identified:

– Safety (C1): The required safety levels have increased lately. For maintenance strategy, it represents conditions to avoid undesirable results, such as accidents, failure, mistakes, and so on. It also refers to the controllability of reducing known threats to an acceptable level, both in terms of personal safety, plant safety, and environmental safety.
– Costs (C2): Maintenance costs must not exceed acceptable limits. Costs differentiate processes and have direct influence on revenues.
– Quality (C3): Proper maintenance management brings better quality of production and costs reduction, while a poor maintenance management of the equipment leads to the breakdown, investment in repair and perhaps replacement, thus translating into higher costs and also can affect other sectors. Poor maintenance management can lead to loss of production and product quality.
– Value added (C4): In the economic area, value added is the difference between the final and initial price to produce a given product. In the maintenance area, value added means all the benefits and returns of maintenance activities. Generally, the higher the value added, the more return will be obtained with a greater effectiveness of maintenance and with less entry of services or products. The most relevant factors are inventories of spare parts, loss of production and identification of failures.
– Feasibility (C5): It is applied to evaluate whether the maintenance strategy is suitable for the system. According to the different requirements of work and techniques for maintenance strategies, the feasibility criteria can be divided in two relevant evaluation factors: by labor, when managers and maintenance staff prefer maintenance strategies that are easy to implement and understand and technique reliability, still under development, condition-based maintenance and predictive maintenance may be inapplicable for some facilities.

Table 3
Aggregated comparisons and priorities of criteria (AHP)

	C1	C2	C3	C4	C5	Priority
C1	1	4.72	0.29	0.18	2.60	14%
C2	0.21	1	0.24	0.18	1.59	6%
C3	3.47	4.22	1	0.48	5.65	32%
C4	5.59	5.59	2.08	1	2.29	41%
C5	0.38	0.63	0.18	0.44	1	7%

Table 4
Priorities of alternatives (AHP)

	C1	C2	C3	C4	C5	Overall
A1	38%	52%	46%	56%	36%	48%
A2	46%	32%	19%	24%	15%	26%
A3	8%	7%	20%	11%	30%	15%
A4	7%	9%	15%	9%	19%	11%
CR	0.09	0.01	0.01	**0.15**	0.06	**0.22**

4.2. Analytic hierarchy process application

The three managers were considered experts of equal importance, but they were individually interviewed for AHP application. Their individual pairwise comparisons matrices were aggregated by geometrical mean (Section 3.1). Table 3 presents an aggregated comparisons matrix for the criteria.

For the matrix in Table 2, $\lambda_{\max} = 5.55$, $CI \approx 0.14$ and $CR \approx 0.12$. Comparisons matrices with $CR > 0.1$ represent an alert that comparisons may not be logically connected. After review in a group meeting, the experts decided to keep the priority vector. Then, Value Added (C4) and Quality (C3) are the highest priority criteria. This result is reasonable considering the high competition in the company's market.

The experts provided individual pairwise comparisons matrices among alternatives regarding to each criterion. Table 4 presents a decision matrix, with overall priorities and the *CR* for every aggregated comparison matrix. C4 had $CR > 0.1$. Overall *CR* was also greater than the upper limit of 0.1. So, the comparisons needed to be reviewed. However, after the decision matrix was presented to the experts, they agreed with the overall priorities, indicating A1 as the best alternative.

Table 5
Aggregated fuzzy comparisons and priorities of criteria

	C1	C2	C3	C4	C5	Priority
C1	[1, 1, 1]	[2.47, 4.72, 5.74]	[0.20, 0.28, 0.69]	[0.13, 0.18, 0.33]	[1, 2.62, 4.22]	18%
C2	[0.20, 0.21, 0.42]	[1, 1, 1]	[0.15, 0.25, 0.25]	[0.15, 0.35, 0.40]	[1.04, 1.71, 2.31]	8%
C3	[3.45, 4, 4.55]	[3.70, 4.17, 4.76]	[1, 1, 1]	[0.34, 0.48, 0.62]	[5.17, 5.74, 6.18]	32%
C4	[5, 5.56, 6.25]	[5, 5.26, 6.25]	[1.61, 2.08, 2.94]	[1, 1, 1]	[2.16, 2.33, 2.47]	35%
C5	[0.33, 0.39, 0.49]	[0.43, 0.58, 0.96]	[0.16, 0.17, 0.19]	[0.40, 0.43, 0.46]	[1, 1, 1]	6%

Table 6
Priorities of alternatives (FAHP)

	C1	C2	C3	C4	C5	Overall
A1	38%	52%	46%	56%	36%	48%
A2	46%	32%	19%	24%	15%	26%
A3	8%	7%	20%	11%	30%	15%
A4	7%	9%	15%	9%	19%	11%

Then, that was a situation: data collected from experts did not pass the consistency check. However, results from this inconsistency were welcomed by experts. They recognized some comparisons were hard to make and they were not sure about them. So, a fuzzy approach was applied.

4.3. Fuzzy analytic hierarchy process application

The experts were invited to fuzzify their comparisons. Then, every previous comparison was assumed as a modal value for a TFS; lower and upper values were added with their review. Table 5 presents the aggregated comparisons matrix among the criteria.

Criteria C3 and C4 still have the highest priorities. Now, their priorities are closer.

Table 6 presents a new decision matrix, with defuzzified local priorities.

Corrective maintenance (A1) still has the highest priority, even with FAHP. The experts reported felt more confident of their data with FAHP than AHP.

4.4. Intuitionistic fuzzy analytic hierarchy process

The comparisons originally provided by the experts were based on AHP Fundamental Scale [8]. Table 7 presents an aggregated matrix of IFS obtained by converting original comparisons with Table 2.

For the matrix in Table 7, $\lambda_{\max} = 5.98$, $CI \approx 0.25$ and $CR \approx 0.22$. Then, with, comparisons needed to be reviewed. However, ordinal priorities are the same as the FAHP application [1–5], and almost the same as the AHP application [1–5]. Cardinally, with IFAHP, priorities are more dispersed: C4 priority increases from 41% to 51%; C5 priority decreases from 7% to 4%.

Table 7
Aggregated defuzzified comparisons and priorities of criteria (IFAHP)

	C1	C2	C3	C4	C5	Priority
C1	1	6.55	0.17	0.14	4.7	13%
C2	0.15	1	0.59	0.14	2.83	7%
C3	5.82	1.68	1	0.35	6.75	26%
C4	7.36	7.36	2.83	1	4.75	51%
C5	0.21	0.35	0.15	0.21	1	4%

Table 8
Priorities of alternatives (IFAHP)

	C1	C2	C3	C4	C5	Priority
A1	35%	53%	51%	64%	40%	55%
A2	53%	33%	18%	21%	12%	25%
A3	7%	6%	21%	10%	31%	12%
A4	6%	8%	11%	6%	17%	8%

Table 8 presents a new decision matrix with IFAHP. As with AHP and FAHP, A1 had the highest priority.

With IFAHP, the overall priority of A1 is over 50%. Therefore as for criteria, priorities of the alternatives are more disperse with IFAHP than AHP and FAHP.

4.5. Hesitant fuzzy analytic hierarchy process application

Table 9 presents an aggregated fuzzy comparisons matrix and priorities of criteria, with HFAHP approach proposed by [18].

For the matrix in Table 9, $\lambda_{\max} = 7.56$, $CI \approx 0.64$and $CR \approx 0.57$. Then, with $CR > 0.1$, comparisons needed to be reviewed. However, ordinal priorities are the same in FAHP and IFAHP applications [1–5]. Cardinally, with HFAHP, priorities are less dispersed than IFAP: the priority of C4 decreases from 51% to 38%; the priority of C5 increases from 4% to 7%.

Table 10 presents a new decision matrix with IFAHP. As with AHP, FAHP and IAHP, A1 had the highest priority.

Table 9
Aggregated fuzzy comparisons and priorities of criteria (HFAHP)

	C1	C2	C3	C4	C5	Priority
C1	[1, 1, 1]	[1, 5, 9]	[1/9, 1/3, 1]	[1/9, 1/5, 1/3]	[1, 3, 5]	14%
C2	[1/9, 1/5, 1]	[1, 1, 1]	[1/7, 1/5, 7]	[1/9, 1/5, 1/3]	[1, 2, 3]	11%
C3	[1, 3, 9]	[1, 5, 9]	[1, 1, 1]	[1/5, 1/3, 1]	[3, 6, 7]	29%
C4	[3, 5, 9]	[3, 5, 7]	[1, 3, 5]	[1, 1, 1]	[1, 2, 5]	38%
C5	[1/5, 1/3, 1]	[1/3, 1/2, 1]	[1/7, 1/6, 1/3]	[1/5, 1/2, 1]	[1, 1, 1]	7%

Table 10
Priorities of alternatives (HFAHP)

	C1	C2	C3	C4	C5	Priority
A1	45%	52%	44%	52%	42%	48%
A2	39%	30%	19%	28%	15%	26%
A3	10%	7%	21%	12%	26%	15%
A4	7%	11%	15%	6%	17%	11%

With IFAHP, the overall priority of A1 is over 50%. Therefore as for criteria, priorities of the alternatives are more disperse with IFAHP than AHP and FAHP.

5. Conclusions

This paper presents the application of four AHP models to maintenance management in a real-world industrial plant. Models include Classical AHP, Fuzzy AHP (FAHP), and the later developed Intuitionistic Fuzzy AHP (IFAHP) and Hesitant Fuzzy AHP (HFAHP). One starting point to apply those different AHP approaches was the difficulty from the experts to provide consistent comparison matrices. Then, some comparison matrices resulted in consistency ratios $CR > 0.1$.

Even then, AHP was completely applied resulting in Corrective Maintenance (A1) as the maintenance strategy with the highest overall priority. Three FAHP approaches were applied, resulting in the same A1 highest overall priority. Therefore, mathematical results were the same with the different AHP applications. Curiously, with IFAHP and HFAHP CR were greater than AHP. For that reason, both researchers and experts were more confident with FAHP than AHP, HFAHP, and IFAHP.

Therefore, this paper's findings are more philosophical than mathematical. Sometimes, people involved with decision-making are more interested in subjective aspects than objective mathematical measures. This is the fuzzy sets spirit. Of course, with only one case study, this finding cannot be generalized. Case studies in other fields of management, engineering, and social science are welcome to compare our findings. These are the main proposal for future research. The conduction of ex-post facto case studies on AHP applications with fuzzy sets theory. It would be very interesting if diverse results come from different models and the same data.

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References

[1] A. Emrouznejad and M. Marra, The state of the art development of AHP (1979–2017): A literature review with a social network analysis, *International Journal of Production Research* **55** (2017), 6653–6675.

[2] L.G. Vargas, An overview of the analytic hierarchy process and its applications, *European Journal of Operational Research* **48** (1990), 2–8.

[3] T.L. Saaty, The analytic hierarchy and analytic network processes for the measurement of intangible criteria and for decision-making, *in: S. Greco, M. Ehrgott and J.R. Figueira, ed., Multiple Criteria Decision Analysis, N. York, Springer* (2016), 363–419.

[4] T.L. Saaty, Measuring the fuzziness of sets, *Journal of Cybernetics* **4** (1974), 53–61.

[5] K. Kulakowski, Notes on order preservation and consistency in AHP, *European Journal of Operational Research* **245** (2015), 333–337.

[6] F. Moussaoui, M. Cherrared, M.A. Kacimic and R. Belarbi, A genetic algorithm to optimize consistency ratio in AHP method for energy performance assessment of residential buildings–Application of top-down and bottom-up approaches in Algerian case study, *Sustainable Cities and Society* **42** (2018), 622–636.

[7] R. Ohta, V.A.P. Salomon and M.B. Silva, Selection of industrial maintenance strategy: Classical AHP and Fuzzy AHP applications, *International Journal of the Analytic Hierarchy Process* **10** (2018), 254–265.

[8] C. Kahraman, B. Oztaysi, S.C. Onar and O. Dogan, Intuitionistic fuzzy originated interval type-2 fuzzy AHP: An application to damless hydroelectric power plants, *International Journal of the Analytic Hierarchy Process* **10** (2018), 266–292.

[9] H. Liu and R.M. Rodriguez, A fuzzy envelope for hesitant fuzzy linguistic term set and its application to multicriteria decision making, *Information Sciences* **258** (2014), 220–238.

[10] P. Mongeon and A. Paul-Hus, The journal coverage of Web of Science and Scopus: A comparative analysis, *Scientometrics* **106** (2016), 213–228.

[11] R.M. Tong and P.P. Bonissone, A linguistic approach to decision-making with fuzzy sets, IEEE Transactions On Systems, Man, *Cybernetics* **10** (1980), 716–723.

[12] C. Kahraman, B. Oztaysi and S.C. Onar, A comprehensive literature review of 50 years of fuzzy set theory, *International Journal of Computational Intelligence Systems* **9** (2016), 3–24.

[13] L.A. Zadeh, Fuzzy sets, *Information and Control* **8** (1965), 338–353.

[14] L.A. Zadeh, The concept of a linguistic variable and its applications to approximate reasoning–I, *Information Sciences* **8** (1975), 199–249.

[15] I. Grattan-Guinness, Fuzzy membership mapped onto intervals and many-valued quantities, *Mathematical Logic Quarterly* **22** (1976), 149–160.

[16] K.T. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems* **20** (1986), 87–96.

[17] R.R. Yager, On the theory of bags, *International Journal of General Systems* **13** (1986), 23–37.

[18] J.M. Garibaldi and T. Ozen, Uncertain fuzzy reasoning: A case study in modelling expert decision making, *IEEE Transactions on Fuzzy Systems* **15** (2010), 16–30.

[19] V. Torra, Hesitant fuzzy sets, *International Journal of Intelligent Systems* **25** (2010), 529–539.

[20] T.L. Saaty, A scaling method for priorities in hierarchical structures, *Journal of Mathematical Psychology* **15** (1977), 234–281.

[21] T.L. Saaty, *The Analytic Hierarchy Process, N. York*, McGraw-Hill 1980.

[22] T.L. Saaty and K. Peniwati *Group Decision Making* Pittsburgh, RWS, 2013.

[23] E.E. Karsak, Fuzzy multiple objective programming framework to prioritize design requirements in quality function deployment, *Computers and Industrial Engineering* **47** (2004), 149–163.

[24] L.K. Chan, H.P. Kao, A. Ng and M.L. Wu, Rating the importance of customer needs in quality function deployment by fuzzy and entropy methods, *International Journal of Production Research* **37** (1999), 2499–2518.

[25] D.-Y. Chang, Applications of the extent analysis method on fuzzy AHP, *European Journal of Operational Research* **95** (1996), 649–655.

[26] C. Kahraman, B. Oztaysi, I.U. Sari and E. Turanoglu, Fuzzy analytic hierarchy process with interval type-2 fuzzy sets, *Knowledge-Based Systems* **59** (2014), 48–57.

[27] E. Bolturk, S.C. Onar, B. Oztaysi, C. Kahraman and K. Goztepe, Multi-attribute warehouse location selection in humanitarian logistics using Hesitant Fuzzy AHP, *International Journal of the Analytic Hierarchy Process* **8** (2016), 271–298.